EMPIRICAL MODE DECOMPOSITION DESCRIPTOR FOR PLANE CLOSED CURVES

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ABSTRACT

Empirical mode decomposition (EMD) developed by Huang et al.[1] is a nonlinear data analysis method for non-stationary real-valued time series. It has been applied extensively in many research areas. Recently, several generalized EMD methods for complex-valued data analysis was proposed [2] [3] .Since a plane closed curve comprises many two-dimensional (2D) space data points, one can imagine that the boundary points of a plane closed curve as a complex data sequence in the complex plane, and make use of the newly developed complex EMD (CEMD) to do further analysis. We have found that we can use CEMD to achieve boundary points noise-reduction of plane closed curves and perform shift-invariant, scale-invariant and rotation-invariant pattern recognition.

1. INTRODUCTION

The empirical mode decomposition has been applied in numerous fields very popularly, such as cosmological analysis in NASA, bio-medical application in Harvard University, speaker recognition in FBI, seismic signal processing in geophysics and many other research areas, since it was invented. The main idea of EMD is to perform decomposition on data sequence and break it into several partitions, so-called Intrinsic Mode Functions (IMFs) and Residual Function (RF) or Trend Function. By inspecting the properties of the IMFs and RF, we can gain more insight into signal of interest and even use IMFs and RF to do further signal processing. However, the original EMD method proposed by Huang et al.[1] mainly focused on real-valued signal and prevents it from being applied in research fields where complex-valued signal analysis is demanded. In order to deal with the insufficiency of traditional EMD method, Tanaka et al.[2] and Gabriel Rilling et al.[3] proposed different algorithms respectively that extend the computing power of the EMD, making it capable of tackling with complex-valued signal. The former method is based on the signal decomposition principles of original EMD and they use the characteristics of analytical signal in complex domain and Hilbert Transform brilliantly to generalize the EMD concept. The latter method basically follows the spirits of traditional one, but contrary to the original decomposition rationale, which is based on the local oscillating characteristics of real-valued signal, considering interested signal as being composed of 1D fast oscillating signals (IMFs) plus 1D slowly oscillating signal (RF), they introduced the concept of viewing complex-valued time series (2D signal) as rotating motion in the three-dimensional space as time passes. In other words, we can now treat complex-valued signal as 2D fast rotating signals (CIMFs) plus 2D slowly rotating signal (CRF).The detailed analysis and explanations of underlying principles about CEMD were specified in [3], the interested readers may refer to it for further informations. The rest of this work is organized as follows. Section 2 discusses the application of CEMD to generate shift, scale and rotation-invariant shape descriptor for plane closed curves, noise-reduction and detection of partial changes in these curves. Detailed formulations and principles are shown. Simulated and experimental results are shown in section 3.Finally, section 4 concludes this paper.

2. APPLICATIONS OF CEMD AND CIMFS

2.1 Transform plane closed curve into complex sequence

Equipped with the CEMD method mentioned above, one can now explore the possibility of its application in areas where complex-valued signal is involved. In this work, we mainly focus on the application of CIMFs as a shape descriptor for plane closed curves which is similar to the fourier descriptor [4]. Making use of the CIMFs as shape descriptor of plane closed curves acquired through image segmentation procedure, we can perform shift, scale and rotation-invariant pattern recognition. The plane closed curve comprises many boundary points which are 2D grids in Cartesian coordinates, and we can denote the coordinates as a complex number. For example, a boundary point at Cartesian coordinates (a,b) can be viewed as a complex number a+bj in the complex plane. Given a segmented plane closed curve, one can transform all the Cartesian coordinates of the boundary points into their counterparts in complex plane first, and then calculate the center of mass of these complex numbers. Secondly, taking the farest point from the center of mass as our point of departure and connecting all the remaining points sequentially, either in clockwise or counterclockwise

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direction. After putting these boundary points all together, we have constructed a complex-valued data sequence which is a representation of the original curve. Finally, through the process of performing CEMD on this data sequence, decomposing it into CIMFs and CRF, we can take these partial signals as shape descriptor which is shift, scale and rotation-invariant to perform pattern recognition or shape matching. And even more, we can also subtract these CIMFs from the original data to reduce the data noise of curve accompanied by sensing process, or detecting partial changes of curve's shape.

2.2 Noise-reduction

Noise is an annoying artifact which affects the perceptual quality and increases the difficulty of pattern recognition of the segmented plane closed curves after sensing and segmentation process. Although there are many noise models available to characterize the noise under different environments and conditions, we choose AWGN model to demonstrate the noise-reduction capability of the CEMD method because it is a very simple yet sufficient model for illustration purpose. Hence, we restrict our experiment to the AWGN model. That is, we model the noise-corrupted data sequence y(n) as the following equation, where x(n) is the ideal boundary data sequence of the plane closed curve and ε(n) is AWGN.

$$y(n) = x(n) + \varepsilon(n) \text{, where } \varepsilon \overset{i.i.d.}{\sim} N(0, \sigma_\varepsilon^2) \quad \textbf{(1)}$$

As mentioned previously, CEMD is a kind of signal decomposition method which decomposes the original data into fast rotating parts (CIMFs) and slowly rotating parts (CRF).Since noise varies randomly and changes very rapidly, we expect that the fast rotating parts (CIMFs) will capture most of the noise movements. By subtracting some of the CIMFs from the noise-corrupted data, we can get a more pleasing, noise-reduced data sequence for further processing. The procedure for noise-reduction appears as :

Step 1 Acquire noise-corrupted data y(n) as (1) shows.
Step 2 Generate the replica y'(n) of the data y(n) in step 1.
Step 3 Concatenate y(n) and y'(n) into a new sequence s(n).
Step 4 Extract one-cycle data z(n) from two-cycle data s(n).
Step 5 Perform CEMD on z(n) and generate its CIMFs&RF.
Step 6 Subtract the first two CIMFs from z(n) to get z'(n).
Step 7 Finally, we obtain a noise-reduced sequence z'(n).

The reason of concatenating original sequence and its replica and performing CEMD on z(n) is that we could avoid boundary effect as Fig.1 shows. Contrary to the fourier descriptor based method [4], which is only capable of reducing global noise, we have performed another experiment that takes use of the mentioned CEMD descriptor based method to reduce the noise of locally

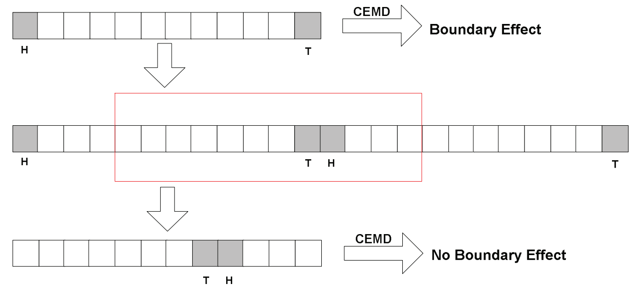

Figure 1 Avoid boundary effect by extracting z(n) from s(n)

noise-corrupted and simulations and other consideration are shown in section 3.

2.3 CIMFs as shape descriptor for pattern recognition

Shift, scale and rotation-invariant properties are the major requirements for doing pattern recognition. When the interested object is not positioned correctly, the image and the following segmented plane closed curves will be shifted. The size of the projection of the object in the focal plane depends on the distance from the camera to the object. For example, as the object approaches the camera, we get a bigger projection in the image plane i.e. the object is scaled. The difference of tilt angle between the object and the image plane will lead to rotation of the segmented plane close curves. In order to overcome these problems, we propose a simple method that takes CIMFs as shape descriptor. By computing cross-correlation function (CCF) between segmented object's CIMFs and database objects' CIMFs, we observe that we can still recognize these objects of interest, even if they have been shifted, scaled or rotated. To analyze the proposed method, we perform geometric transformations (i.e. shift, scaling and rotation) on some selected objects from database and then compute the CCF between these choosed objects and database ones. The transformation of coordinates is characterized as:

$$(s,t) = T\{(x,y)\} \quad \textbf{(2)}$$

or one can use affine transform which has matrix form as:

$$\begin{bmatrix} s & t & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \mathrm{T} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix} \quad \textbf{(3)}$$

The coordinate equations and Affine transformation matrix T of shift, scaling and rotation are summarized as follows:

$$\mathit{Shift} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ v_s & v_t & 1 \end{bmatrix} \qquad \begin{aligned} s &= x + v_s \\ t &= y + v_t \end{aligned}$$

$$\text{Scaling} \quad \begin{bmatrix} k_s & 0 & 0 \\ 0 & k_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} s &= k_s x \\ t &= k_t y \end{aligned}$$

$$\text{Rotation} \quad \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} s &= x\cos\theta - y\sin\theta \\ t &= x\sin\theta + y\cos\theta \end{aligned}$$

Above matrices for shift, scaling &rotation are from [5][6]. The procedure for taking CIMFs as shape descriptor and finding object in the database that the shape is closest to original one appears as follows:

Step 1 Acquire the transformed data y(n) form input x(n).
Step 2 Generate the replica y'(n) of the data y(n) in step 1.
Step 3 Concatenate y(n) and y'(n) into a new sequence s(n).
Step 4 Extract one-cycle data z(n) from two-cycle data s(n).
Step 5 Perform CEMD on z(n) and generate its CIMFs&RF.
Step 6 Compute CCF of CIMF1 between z(n) and database object,CIMF1 stands for 1st CIMF and denote CCF as R(τ).
Step 7 Find object j in database that maximizes R(0) (dc term of CCF).The CCF is cross-correlation function which is defined as : $R_{xy}(m) = E\{x_{n+m} y_n^*\}$ **(4)**

3. EXPERIMENTAL RESULTS

The testing plane closed curves of this work are all from the MPEG-7 CE1-part B database which is designed to evaluate similarity retrieval.Fig.2 demonstrates six kinds of testing shape objects for illustration purpose.

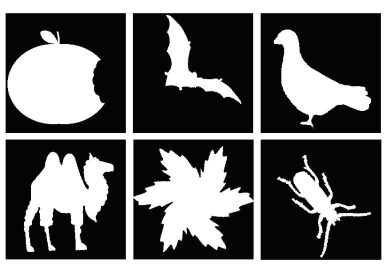

Figure 2 Testing shape objects.

3.1 Experimental results about noise-reduction

As mentioned in section 2.2, we can subtract CIMFs (e.g. the first two CIMFs)from globally noise-corrupted or locally noise-corrupted data z(n) which is obtained from plane closed curve to produce a noise-reduced version of data.Fig.3 shows the simulated results which are generated by MATLAB R2008a.The program is also written in MATLAB language.Fig.3(a)(d) shows coordinates of boundary data points of plane closed curves(as Fig.2 depicts) in complex plane (horizontal-axis stands for imaginary-axis, vertical-axis stands for real-axis, the origins are positioned at top left corner of these six sub-figures).

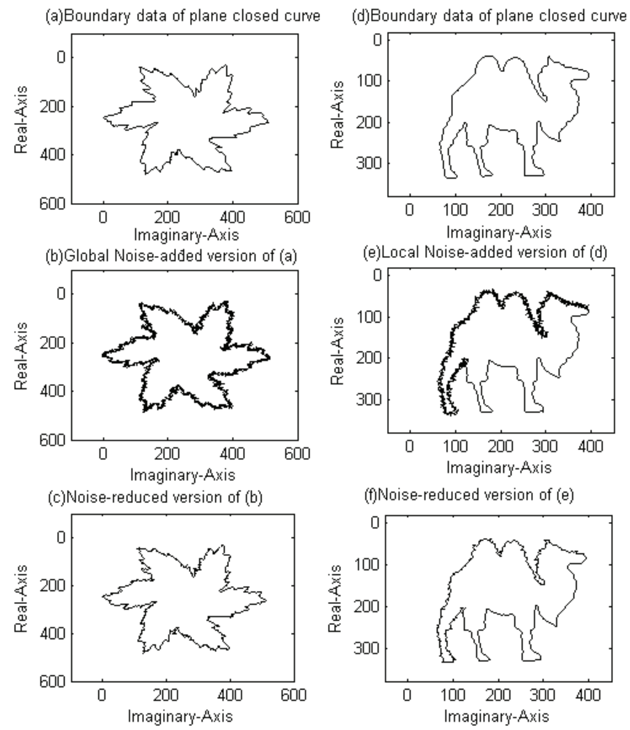

Figure 3 Simulated results of the noise-reduction procedure.

Fig.3 (b) (e) shows noise-corrupted boundary points where the left column is globally noise-corrupted version and the right column is locally noise-corrupted version. The noise is modeled as AWGN and its variance is 3.Finally, Fig.3(c)(f) shows noise-reduced data. As one can observe from these figures that the noisy disturbances are successfully reduced by subtraction process of the first two CIMFs.

3.2 Experimental results on CIMFs as descriptor

We illustrate several examples of using CIMFs as shape descriptor. First experiment is about scaling and shift as Fig.4 shows.The left column of Fig.4 shows the shape 'bat-7',its 2x scaled version and CCF between their CIMFs. The right column shows the shape 'bat-7', its shifted version (real-axis:152 pixels imaginary-axis:184 pixels),and CCF between their CIMFs. We observe that the CCFs all have strong peaks at |R(0)|, indicating that the shapes are similar. The peak under 2x scaled situation is two times higher than that under shifted situation, indicating that the energy of scaled version is two times higher than shifted version. The left column of Fig.5 depicted the shape 'bird-17',its +60 degrees(counterclockwise) rotated,2x scaled and shifted (real-axis:152 pixels imaginary-axis:184 pixels) version of data and the CCF between their CIMF1.It is clear that there is a strong peak at dc,i.e.|R(0)|, demonstrating high correspondence between the object 'bird-17'and its transformed version (after three operations). On the other

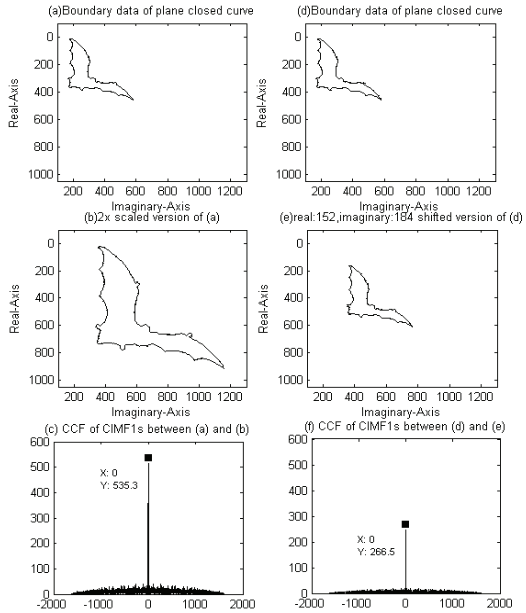

Figure 4 CIMFs as shape descriptor (scale and shift)

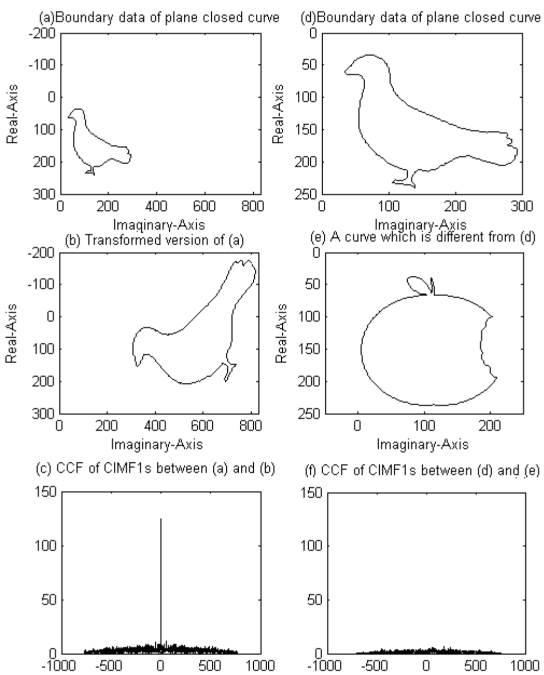

Figure 5 CIMFs as shape descriptor (rotation, scale & shift)

hand, by inspecting the right column of Fig.5,we observe that the CCF between two different shapes ('bird-17' and 'apple-9') does not have a peak at |R(0)| or at remaining lag indices 'm'. According to the discussions in the above paragraph, we can take CIMFs as a simple shape descriptor by computing CCF between first CIMFs of different shapes and compare the value at |R(0)| to classify these different shapes. Finally, CIMFs may also be used to detect partial changes of curve's shape. As Fig.6 depicts, the first row shows boundary points of the shape object 'bug'. The right

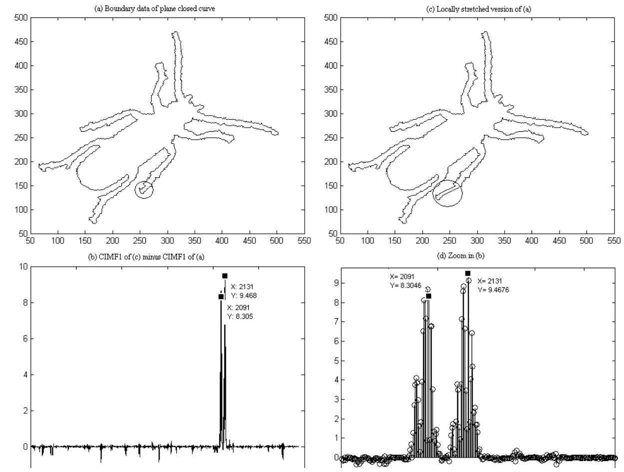

Figure 6 Use CIMFs to detect partial changes

plot is almost equivalent to the left one except one of the bug's legs is stretched. The second row shows the difference between their first CIMFs. The right column is zoomed version of the left one. We observe that there are peaks at the beginning and ending positions of the stretched part and differences are almost zero at other positions, indicating that we may use CIMFs to detect small partial changes between almost the same curves. The execution time that is necessary to generate above descriptors is about 5 secs (CPU:Intel I7-920 quad core,RAM:3GB DDR3 1333MHz).

4. CONCLUSION

Several applications of CEMD about noise-reduction, shift, scale and rotation-invariant shape description of plane closed curves and detection of their shape's partial changes have been introduced. The experimental results about these applications are also demonstrated.

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